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## Optimizing Public Transport Systems in Sub-Saharan Africa using Operational Research Technique: a Focus on Nigeria

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### Abstract

The rapid urbanization of sub-Saharan Africa metropolitan areas combined with inadequate or poorly executed development plans, has given rise to numerous transportation problems, including deteriorated physical attractions and comfort of road based public transport. This addresses these problems. Mathematical model is used to optimize the public transport system of Lagos, Nigeria. Lagos being a typical sub-Saharan metropolitan city, is used as a good representation of others. The public transport system in Lagos is modelled using linear programming algorithm. The resulting linear programming model was solved using simplex method, with the aid of a computer software – LIP solver. The optimal transport system, if implemented, will eradicate the problem caused by poor public transport system in sub-Saharan Africa or at least reduce them to the barest minimum.

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### 1. Introduction

Sub-Saharan African countries are often referred to as the “Black” Africa and considered more authentically “Africa”. The choice to focus on Nigeria in this paper is because Nigeria is a good representation of sub-Saharan Africa. Lagos on the other hand can be referred to as a small Nigeria, because every part of Nigeria is adequately represented in Lagos. Sub-Saharan countries include: Senegal, Mali, Niger, Chad, Eritrea, Djibouti, Ethiopia, Somalia and all other countries below them, up to South Africa, including Madagascar, as we can see in figure 1. Africa is, geographically, the area of the continent of Africa that lies south of the Saharan Desert. Politically, it consists of all African countries that are fully or partially located south of the Sahara (excluding Sudan, even though Sudan sits in the Eastern portion of the Sahara Desert) [2]. Somalia, Djibouti, Comoros and Mauritania are however geographically part of sub-Saharan Africa, but also part of the Arab world (from Wikipedia, the free encyclopaedia).

Effective movement of people, goods and information through places in nation is very important as it affects the economic, political, social and cultural life of such nation by performing the role of linking supply and demand [6,8].

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Transportation links between countries make the socio-political relationship between them possible. Importation and exportation of goods and services depend so much on the provider of logistics and movement of the goods imported or for export [7,8,10]. Transportation is the life wire of any urban society such as Lagos, Nigeria. Lagos just like any other fast growing urban city has transportation problems, ranging from congestion to air pollution and traffic accidents [2]. The objective of this paper is to minimize the causes of the problems of public road transportation system in the urban cities of sub-Saharan Africa countries, using Lagos as a case study. Most scholars, who worked on urban transportation problems in Nigeria, have identified congestion as the most serious. Congestion occurs when transport demand exceeds transport supply at a specific point in time and in a specific section of the transport systems [1]. Lagos as a commercial and industrial nerve centre of Nigeria has the most problem of traffic congestion. The causes of this congestion can be traced to the following: structural pattern of the roads and the unplanned growth and haphazard land-use distribution. Associated with the problem of packing, accidents, environmental pollution and noise pollution. Energy consumption and land consumption are on the increase due to the increase in the number of vehicles on the road at a particular time.



Fig. 1. Chaotic Road Transportation in Lagos, Nigeria

In order to reduce congestion, which is the bane of most of the urban transport problems, the following factors have to be maximized: Good drainage facilities; Provision of off-street parking facilities; Provision of traffic light; Creation of barriers, overhead foot bridges or under passes; Good road maintenance culture; Comprehensive transportation study; Linear programming, or Operational research technique is used in this paper to maximize these factors in order to minimize the identified urban transport problems caused basically by congestion in urban cities [4].

Operational research techniques are methods of mathematically based analysis for providing a quantitative basis for management decisions [3,9]. Therefore minimizing the cost of achieving better transportation system by maximizing those factors mentioned above is the main thing for the management.

## 2. Formulation of Problem

Let the factors responsible for congestion in the urban areas of sub-Saharan African are:

- Drainage Facilities
- Provision of off street Parking Facilities
- Provision of traffic light
- Creation of Barriers
- Overhead Foot Bridges or Under Passes
- Good Road Maintenance Culture
- Comprehensive transportation study

In formulating the problem, the above factors are taking to be our decision variables( $x_{ij}$ ).

The objective of this paper is to maximize public road transport system in Lagos Nigeria.

### 2.1. Decision Variables

Let  $x_{ij}$  represent number of factor  $i$  needed to use up  $j$  resource in order to maximize public road transport system in Lagos.

$x_{1j}$  = Good Drainage Facilities

$x_{2j}$  = Provision of off street Parking Facilities

$x_{3j}$  = Provision of traffic light

$x_{4j}$  = Creation of Barriers, Overhead Foot Bridges or Under Passes

$x_{5j}$  = Good Road Maintenance Culture

$x_{6j}$  = Comprehensive transportation study

$C_j$  represents the contribution to  $x_{ij}$ .

Let

$x_{11}$  represent the number of good drainage facilities to be provided within a year.

$x_{12}$  represent the number of drainages to be maintained in the last one year.

$x_{21}$  represent the number of off street Parking Facilities to be provided within a year

$x_{22}$  represent the number of times these off street Parking Facilities should be maintained within a year.

$x_{31}$  represent the number of traffic lights to be provided.

$x_{32}$  represent the number of times they should be maintained within a year.

$x_{41}$  represent the number of Overhead Foot Bridges or Under Passes that should be provided within a year.

$x_{42}$  represent the number of times they should be maintained after construction.

$x_{51}$  represent the number of times they should be maintained in a year.

$x_{52}$  represent the number of times they should be maintained in a year

$x_{61}$  represent the number of times in a year to embark on transportation study.

$x_{62}$  represent the number of roads comprehensive transportation study should be carried out

From the data gathered, the contribution  $C_j$ , as regards the different factors (decision variables)  $x_{ij}$  are represented as follows:

$C_1 = 0.3$ ,  $C_2 = 0.2$ ,  $C_3 = 0.2$ ,  $C_4 = 0.8$ ,  $C_5 = 0.2$ ,  $C_6 = 0.8$ ,  $C_7 = 0.4$ ,  $C_8 = 0.6$ ,  $C_9 = 0.5$ ,  $C_{10} = 0.5$ ,  $C_{11} = 0.5$ ,  $C_{12} = 0.5$ .

### 2.2. Limitations to solving congestion problems

(i) Fully eradication of traffic is not affordable and possible especially in economically dynamic urban areas such as Lagos.

(ii) Expansion of road infrastructure is expensive and has wide – ranging economic, social and environmental effects.

(iii) Selection of a strategic “Mix” of measures that will be effective in solving traffic congestion is not an easy task.

### 2.3. Sources of Congestion (in %)

From investigation carried out, the following are the sources of congestion in percentages

Bottlenecks – 40%

Traffic incidents – 25%

Work zones – 10%

Bad weather – 15%

Poor Signal Timing – 5%

Specific Events/others – 5%

### 2.4. Resources per Unit of a decision Variable ( $a_{ij}$ )

Let  $a_{11}$  represent the unit cost of providing Drainage Facility

Let  $a_{12}$  represent the cost of maintaining the drainage facility.  
 Let  $a_{21}$  represent the unit cost of providing off- street parking facility  
 Let  $a_{22}$  represent the unit cost of maintaining off- street parking facility.  
 Let  $a_{31}$  represent the cost of providing one unit of traffic light  
 Let  $a_{32}$  represent the cost of maintaining one unit of traffic light.  
 Let  $a_{41}$  represent the cost of providing one unit of road barriers, overhead foot bridges, or under passes.  
 Let  $a_{42}$  represent the cost of maintaining one unit of road barriers, overhead foot bridges, or under passes.  
 Let  $a_{51}$  represent the unit cost of achieving road maintenance culture.  
 Let  $a_{52}$  represent the unit cost of continuing in that culture.  
 Let  $a_{61}$  represent the unit cost of embarking on comprehensive transportation study.  
 Let  $a_{62}$  represent the unit cost of embarking on comprehensive transportation study per road.

## 2.5. Available Recourses ( $b_i$ )

Let  $b_1$  represent the amount available for providing good drainage facilities.  
 Let  $b_2$  represent the amount available for provision of off- street parking facility  
 Let  $b_3$  represent the amount available for providing of traffic light  
 Let  $b_4$  represent the amount available for creation of road barriers, overhead foot bridges, or under passes.  
 Let  $b_5$  represent the amount available for providing good road maintenance culture.  
 Let  $b_6$  represent the amount available for carrying out comprehensive transportation study.

### 2.5.1. Utilization of the available Recourses for solving congestion problems

From the data gathered and subsequent computations, the following represent some of the values obtained:

$$b_1 = 15\% \text{ of } \sum b_i, b_2 = 5\% \text{ of } \sum b_i, b_3 = 8\% \text{ of } \sum b_i, b_4 = 50\% \text{ of } \sum b_i, b_5 = 12\% \text{ of } \sum b_i, b_6 = 10\% \text{ of } \sum b_i$$

The total available resources, in terms of money is assumed to be N20 billion.

That is  $\sum_{i=1}^n b_i = \text{N20 billion}$ .

Similarly,  $a_{11} = 10,000, a_{12} = 5000, a_{21} = 6000, a_{22} = 3000, a_{31} = 8000, a_{32} = 3000, a_{41} = 2000, a_{42} = 10000, a_{51} = 4000, a_{52} = 3000, a_{61} = 6000, a_{62} = 4000$

## 2.6. Constraints

$$a_{11}x_{11} + a_{12}x_{12} \leq b_1$$

$$a_{21}x_{21} + a_{22}x_{22} \leq b_2$$

$$a_{31}x_{31} + a_{32}x_{32} \leq b_3$$

$$a_{41}x_{41} + a_{42}x_{42} \leq b_4$$

$$a_{51}x_{51} + a_{52}x_{52} \leq b_5$$

$$a_{61}x_{61} + a_{62}x_{62} \leq b_6$$

$$a_{11}x_{11} + a_{12}x_{12} + a_{21}x_{21} + a_{22}x_{22} + a_{31}x_{31} + a_{32}x_{32} + a_{41}x_{41} + a_{42}x_{42} + a_{51}x_{51} + a_{52}x_{52} + a_{61}x_{61} + a_{62}x_{62} \leq \sum_{i=1}^n b_i$$

$$x_{ij} \geq 0, i = 1, 2, \dots, 6, j = 1, 2,$$

## 2.7. The Model

$$\text{Maximize } Z = 0.3x_{11} + 0.7x_{12} + 0.2x_{21} + 0.8x_{22} + 0.2x_{31} + 0.8x_{32} + 0.4x_{41} + 0.6x_{42} + 0.5x_{51} + 0.5x_{52} + 0.5x_{61} + 0.5x_{62}$$

**Subject to**

$$0.01x_{11} + 0.005x_{12} \leq 300$$

$$0.006x_{21} + 0.003x_{22} \leq 1000$$

$$0.008x_{31} + 0.003x_{32} \leq 1600$$

$$0.02 + 0.01x_{42} \leq 10,000$$

$$\begin{aligned}
&0.004x_{51} + 0.003x_{52} \leq 2,400 \\
&0.006x_{61} + 0.003x_{62} \leq 2000 \\
&0.01x_{11} + 0.005x_{12} + 0.006x_{21} + 0.003x_{22} + 0.008x_{31} + 0.003x_{32} + 0.02 + 0.01x_{42} + 0.004x_{51} + 0.003x_{52} \\
&\quad + 0.006x_{61} + 0.003x_{62} \leq 20,000 \\
&x_{ij} \geq 0, i = 1,2, \dots, 6, j = 1,2,
\end{aligned}$$

### 3. Model Solution

#### 3.1. Standardized Model

Maximize

$$\begin{aligned}
Z = &0.3x_{11} + 0.7x_{12} + 0.2x_{21} + 0.8x_{22} + 0.2x_{31} + 0.8x_{32} + 0.4x_{41} + 0.6x_{42} + 0.5x_{51} + 0.5x_{52} + 0.5x_{61} \\
&\quad + 0.5x_{62}
\end{aligned}$$

Subject to

$$\begin{aligned}
&0.01x_{11} + 0.005x_{12} + S_1 \leq 300 \\
&0.006x_{21} + 0.003x_{22} + S_2 \leq 1000 \\
&0.008x_{31} + 0.003x_{32} + S_3 \leq 1600 \\
&0.02 + 0.01x_{42} + S_4 \leq 10,000 \\
&0.004x_{51} + 0.003x_{52} + S_5 \leq 2,400 \\
&0.006x_{61} + 0.003x_{62} + S_6 \leq 2000 \\
&0.01x_{11} + 0.005x_{12} + 0.006x_{21} + 0.003x_{22} + 0.008x_{31} + 0.003x_{32} + 0.02 + 0.01x_{42} + 0.004x_{51} + 0.003x_{52} \\
&\quad + 0.006x_{61} + 0.003x_{62} + S_7 = 20,000 \\
&x_{ij} \geq 0, i = 1,2, \dots, 6, j = 1,2,
\end{aligned}$$

#### 3.2. 3.2 Simplex Method of Solution

In this paper, Simplex method algorithm was adopted to solve the standardized Linear Programming model. Computer software was used to evaluate the initial tableau formed from the standardized model. The optimal solution was found as  $1.98533e+006$  after eight iterations with the following decision and slack variables values:

$X_{11} = 0$ ,  $X_{12} = 60000$ ,  $X_{21} = 0$ ,  $X_{22} = 333333$ ,  $X_{31} = 0$ ,  $X_{32} = 533333$ ,  $X_{41} = 0$ ,  $X_{42} = 1e+006$ ,  $X_{51} = 0$ ,  $X_{52} = 800000$ ,  $X_{61} = 0$ ,  $X_{62} = 500000$ ,  $S_1 = 0$ ,  $S_2 = 0$ ,  $S_3 = 0$ ,  $S_4 = 0$ ,  $S_5 = 0$ ,  $S_6 = 0$ ,  $S_7 = 0$ .

### 4. Result Discussions

From table 2 above, the feasible solutions of the decision and slack variables are shown. The following decision variables are very significant in achieving our objective of maximizing public transport systems in sub-Saharan African countries:  $x_{12}$ ,  $x_{22}$ ,  $x_{32}$ ,  $x_{42}$ ,  $x_{52}$ ,  $x_{62}$ , representing; the number of drainages to be maintained in the last one year, the number of times these off street Parking Facilities should be maintained within a year, the number of times they should be maintained within a year, the number of times they should be maintained after construction, the number of times they should be maintained in a year, the number of roads comprehensive transportation study should be carried out, respectively. From the values of these variables it shows that  $x_{42}$  is equal to one million, which means that frequent maintenance should be carried out in order to keep the foot bridges and underpasses in good shape.  $x_{52}$  has value eight hundred thousand. This shows how important maintenance of the general transport infrastructure is in order to maximize public transportation system in the region under consideration. From the results obtained,  $x_{32} = 533,333$  means that the traffic light provided should be maintained at least 533,333 times in a year. Also  $x_{62} = 500,000$  indicates that a comprehensive transportation study should be carried out on at least 500,000 of the roads every year. In order to achieve the objective. In the same vain  $x_{22} = 333,333$  shows the number of times the off-street parking facilities should be maintained within a year. Lastly  $x_{12} = 60,000$  signifies number of times in a year the drainage facilities are to be properly maintained.

## 5. Conclusion

Interest is on optimization of public transport systems in sub-Saharan Africa. The analysis is carried out using simplex method with the aid of computer software (LIPS). The peculiar situation in Lagos Nigeria was modelled as a linear programming problem. It is shown that six out of the twelve decision variables, namely  $x_{12}$ ,  $x_{22}$ ,  $x_{32}$ ,  $x_{42}$ ,  $x_{52}$ ,  $x_{62}$ , are very significant if the objective is to be achieved. Generally, it was shown from the results obtained that maintenance of all the facilities is a must if sub-Saharan Africa countries are serious about optimizing their public transportation systems. Specifically the foot bridges and underpasses should be constantly maintained in order to human traffic on the roads.

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